

# **The Neutron Inner Orbit Model**

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## **Abstract**

The principal postulate for this analysis is that the neutron is actually a proton with an electron in an inner orbit near to the fully contracted proton. It was shown in Ref. 1 that the postulated physical models for the electron and proton are outer shells of C2 particles pulsating 180° out of phase. The dynamic mechanisms associated with these electron and proton models for an inner electron orbit include special relativity, quantization by inverse integers, the modified Coulomb's law, and shared C2 particles (shared mass equivalent energy (mee)) between the proton and electron. These effects are expressed in the appropriate governing equations which are the neutron energy equation, the electron force balance equation, the geometric relation between the descriptive radii, and the electron angular momentum equation. The simultaneous solution of these equations for the special case which utilizes the experimentally determined neutron excess mee as input, results in a calculated orbit that is the closest possible orbit to the proton without exceeding the speed of light for the electron orbital velocity. The solution of the governing equations for the general case which utilizes the orbital index number as input, results in a calculated value of the neutron excess mee that matches the experimental value for the same orbital index number.

## **Introduction**

The analyses reported in Ref. 1 resulted in physical models for the electron and proton that facilitated the emergence of properties of two fundamental massless particles and new properties of the electron and proton. These new relationships accurately calculated the values of the fundamental constants and properties of the electron and proton. This work also suggested the existence of a compatible neutron composed of the new pulsating proton model and the new electron model pulsating 180° out-of-phase with the proton and occupying an inner orbit close to the fully contracted proton. This potential neutron model also raised the possibility of several different physical mechanisms operating in harmonic synchrony. These possible mechanisms for the neutron inner orbit model are based on the following postulates:

1. Single protons possess inner orbits in addition to the outer orbits of the Bohr model of the hydrogen atom.
2. The inner orbits are inside of the Bohr radius with values of angular momentum quantized by inverse integers.
3. The neutron is a proton with an associated electron in an inner orbit close to the fully contracted proton shell.
4. The inner orbit electron shares C2 particles, which is shared mass equivalent energy (mee), with the proton during each expansion cycle.
5. The orbital speed of the inner orbit electron is near the speed of light and therefore the electron is relativistic.
6. The excess mass equivalent energy (mee) of the neutron is equal to the relativistic mee of the electron minus the mee shared between the proton and the electron.

7. Coulomb's law must be modified for the inner orbits nearest the proton by changing the point source for the electrostatic force to a source located at the fully contracted electron or proton shell.

Each of these postulates is explained further in the following paragraphs.

- 1. Single protons possess inner orbits in addition to the outer orbits of the Bohr model of the hydrogen atom.**

The ground state for the Bohr model known as the Bohr radius corresponds to the orbital location nearest to the proton. The radius is calculated to be  $r_1 = 5.29177211(93) \times 10^{-11}$  m where the subscript "1" indicates the first orbit. The radius of the proton shell is calculated to be  $r_{pi} = 1.2685188271(13) \times 10^{-15}$  m. The ratio of these two radii is  $41.7 \times 10^3$ . This means that if the proton radius was one meter, the nearest electron in the Bohr model would be located 41.7 km (25.9 mi) away. This ground state is the only stable orbit for the single electron. The quantized orbits outside of the Bohr radius are all unstable in that they can hold the electron for only a brief period of time in the absence of continuous external energy input. This postulate considers the possibility that there is also a series of unstable quantized orbits between the Bohr radius and the proton radius.

- 2. The inner orbits are inside of the Bohr radius with values of angular momentum quantized by inverse integers.**

The rationale for considering the quantization of inner orbit angular momentum by inverse integers is based on the supposition that quantization is founded in causality. The basis for quantization may be correspondence to beats in a characteristic frequency of the universe. The characteristic frequency may be the pulsation frequency of the electron and proton<sup>2</sup>. The beats, as in music, represent the integer or inverse integer multiple of the characteristic frequency that reinforces the frequency.

For the electron orbital case, the orbits correspond to angular momentum levels which may, in turn, correspond to the differences in the number of shared C2 particles between the orbits. It is these packets of C2 particles that may be the result of the specific harmonic beat frequencies.

- 3. The neutron is a proton with an associated electron in an inner orbit close to the fully contracted proton shell.**

It is known that the free neutron is unstable, decaying into a proton, a relativistic electron, and an antineutrino in approximately an average of 14.8 minutes. A small percentage of the decays produce a gamma ray, and likewise, a small percentage produce a hydrogen atom. It is also generally believed that the proton, electron, and antineutrino are actually created during the decay process. This postulate considers the possibility that the neutron is composed of a proton and a relativistic electron in an inner orbit. Since the free neutron possesses excess mass equivalent energy (mee), it gives up that energy spontaneously requiring no external energy to ionize the proton. In contrast, all the elements possess deficit mee requiring ionization energies from an external source.

The range of decay times for free neutrinos is significant, possibility depending on the state of the pulsating proton and electron at the initiation of the decay process. This would be the initial condition at the moment of ejection of the neutron from its parent atom. The decay process would be expected to be slow because the electron experiences a balance of the electrostatic force and the centripetal force. However, the pulsating proton and electron create a slowly developing dynamic instability that will result in the electron's escape from its inner orbit. It is hypothesized that for most initial conditions, the relativistic electron has sufficient kinetic energy to escape the proton. For these cases, some portion of the relativistic mee may escape from the vicinity of the electron as an antineutrino. For some small percentage of cases the electron may momentarily occupy another inner orbit further away from the proton resulting in the generation of a gamma ray. Likewise, also in a small percentage of cases, a sufficiently large percentage of the relativistic mee may escape from the vicinity of the electron to allow the electron to occupy the ground state and form the hydrogen atom.

**4. The inner orbit electron shares C2 particles, which is shared mass equivalent energy (mee), with the proton during each expansion cycle.**

The concept of the electron and proton as pulsating structures composed of specific numbers of C2 particles<sup>1</sup> provides a potential mechanism for the sharing of C2 particles during their expansion phases. The shells of the structures only require the precise number of C2 particles during their contraction phases. Since the electron and proton pulsate 180 degrees out of phase, they can share the particles and fulfill the number requirement. The shared mee between a proton and its companion electron for both inner and outer (Bohr) orbits is equal to the amount of energy that is required to remove the electron from orbit which is the ionization energy. The interpretation of this relationship is that the shared C2 particles binds the electron to the proton during the electron's expansion phase. In order for the electron to escape, the shared C2 particles must be replaced.

For orbiting electrons, this mee is equal to the electron's orbital kinetic energy. This process is analogous to an orbiting body in a gravitational field where the additional energy required to escape is equal to the body's orbital kinetic energy. A principal difference is that the required energy in the gravitational case comes from additional velocity provided by thrust whereas the required energy in the electron case comes from additional mee. The calculated fraction of the electron mee shared with the proton is small for the Bohr radius orbit and decreases with increasing orbit radius. The ionization energy for all outer orbits must be supplied externally since the bound electrons create a mee deficit. In contrast, the free neutron has a mee excess in the form of the electron associated relativistic mee. This energy is transferred as C2 particles in the decay process to replace the electron's shared C2 particles.

**5. The orbital speed of the inner orbit electron is near the speed of light and therefore the electron is relativistic.**

The balance of the electrostatic force with the centripetal force requires an increasing orbital speed as the orbit radius decreases. This results in an increasing electron kinetic energy with a proportional increase in shared mee. The limiting case is an orbital speed equal to the speed of

light which corresponds to the number of shared C2 particles equal to one-half of the electron's constant number of C2 particles. This means that the shared mee is equal to one-half of the electron mee which does not include the accompanying but separate relativistic mee. The concept of accompanying but separate relativistic mass, energy, and linear momentum associated with an electron or proton moving relative to a stationary C2 particle grid is presented in detail in Ref. 2.

**6. The excess mass equivalent energy (mee) of the neutron is equal to the relativistic mee of the electron minus the mee shared between the proton and the electron.**

The excess mee of the neutron is calculated as the difference between the mass equivalent rest energy of the neutron and the sum of mass equivalent rest energies of its components. In the neutron decay process, a portion of the relativistic mee associated with the electron is transferred to the electron to replace the mee shared with the proton. The remaining relativistic mee is the excess mee of the neutron.

**7. Coulomb's law must be modified for the inner orbits nearest the proton by changing the point source for the electrostatic force to a source located at the fully contracted electron or proton shell.**

Coulomb's law assumes that the electrostatic force results from the influence of point charges. In this postulated model the charge emanates from the spherical surface of both the fully contracted electron and proton. Coulomb's assumption results in highly accurate predictions for cases where the separation distance is much greater than the fully contracted electron and proton radius which has been evaluated<sup>1</sup> as  $r_{ei} = r_{pi} = 1.2685188271(13) \times 10^{-15}$  m. The point source assumption is accurate even for the Bohr model of the hydrogen atom where the Bohr radius is calculated to be  $r_1 = 5.29177211(93) \times 10^{-11}$  m. At this distance, the relative standard uncertainty associated with the point source assumption is  $4.8 \times 10^{-5}$  which is less than five-thousandth of one percent.

However, in the postulated model of the neutron, the time-averaged radius of the associated electron orbit is close to the time-averaged fully contracted proton shell radius. At this distance, the point source assumption is not accurate. In the near field of the pulsating electron or proton, the C1 particles expand from the open areas between the captive C2 particles. This means that in the near field, Coulomb's law should be modified by using the distance of the source as the time-averaged radius of the orbiting electron minus the radius of the fully contracted proton. Since the distance squared occurs in the denominator of the equation, the electrostatic force between the electron and proton becomes large in the near field.

**Special Case for the Neutron Excess Mass Equivalent Energy**

The following analysis is the special case for which the experimentally determined neutron excess mass equivalent energy (mee) is an input to the neutron energy equation. The objective is to formulate the equations that mathematically describe the neutron's physical state and solve these equations for the corresponding neutron properties. These equations are the neutron energy equation, the electron force balance equation, the geometric relation between the descriptive radii, and the electron angular momentum equation. The neutron energy equation is solved for

the electron orbital speed which becomes an input to the equation for the force balance between the electron centripetal force and the proton electrostatic force. This force balance equation is solved simultaneously with the geometric equation to obtain the orbital radius and the electrostatic force radius. The electron angular momentum equation is then solved for the orbital index number. If the above postulates represent reality, it is expected that these calculated neutron properties corresponding to this experimentally determined energy level will be physically meaningful.

The analysis assumes a circular inner orbit for the electron around the proton. The uncertainties associated with this assumption will be estimated and included in the final analysis. As noted above, the excess mass of the neutron is calculated as the difference between the mass equivalent rest energy of the neutron and the sum of mass equivalent rest energies of its components.

The neutron excess mass  $\nabla m_n$  is the difference between the neutron mass  $m_n$  and the sum of the proton mass  $m_p$  and the electron mass  $m_e$  yielding

$$\nabla m_n = m_n - m_p - m_e = 1.39464(15) \times 10^{-30} \text{ kg} \quad (1)$$

The neutron excess mass becomes

$$\nabla m_n c^2 = 1.25344(14) \times 10^{-13} \text{ J} \quad (2)$$

The kinetic energy of the electron plus the associated relativistic mass, both for the  $n^{\text{th}}$  orbit, is given by

$$\frac{1}{2} m_e V_{eno}^2 + m_{nR} c^2 = \frac{m_e c^2}{\sqrt{1 - \frac{V_{eno}^2}{c^2}}} - m_e c^2 \quad (3)$$

The  $V_{eno}$  is the electron orbital speed for the  $n^{\text{th}}$  orbit,  $n$  is the orbital index number,  $m_{nR}$  is the relativistic mass for the  $n^{\text{th}}$  orbit, and  $c$  is the speed of light. The concept of a separate relativistic mass that accompanies the constant mass electron is presented in Ref. 2. The first term on the right hand side of Equation (3) is the total energy of the electron plus the associated relativistic mass. The second term on the right hand side is the electron rest energy. It was shown in Ref. 2 that by using the binomial expansion for the square root term, the right hand side approaches the electron kinetic energy as  $V_{eno}$  approaches zero. It was also shown in Ref. 2 that  $m_{nR}$  approaches zero as  $V_{eno}$  approaches zero. The result is that as  $V_{eno}$  approaches zero, Equation (3) approaches

$$\frac{1}{2} m_e V_{eno}^2 = \frac{1}{2} m_e V_{eno}^2 \quad (4)$$

This exercise demonstrates that the relativistic energy equation reduces to the expected expression for the electron kinetic energy at low velocities.

Dividing both sides of Equation (3) by  $m_e c^2$  and introducing  $\beta_n = \frac{V_{eno}^2}{c^2}$ , the resulting equation becomes

$$\frac{1}{2}\beta_n + \frac{m_{nR}}{m_e} = \frac{1}{\sqrt{1-\beta_n}} - 1 \quad (5)$$

Postulate No. 6 above states that the excess mass equivalent energy (mee) of the neutron is equal to the relativistic mee of the electron minus the mee shared between the proton and the electron. Postulate No. 4 above states that the inner orbit electron shares C2 particles, which is shared mass equivalent energy, with the proton during each expansion cycle. For orbiting electrons, this mee is equal to the electron's orbital kinetic energy. In equation form, these statements become

$$\nabla m_n c^2 = m_{nR} c^2 - \frac{1}{2} m_e V_{eno}^2 = 1.25344(14) \times 10^{-13} \text{ j} \quad (6)$$

Dividing this entire equation by  $m_e c^2$  to obtain the normalized neutron excess mee yields

$$\frac{\nabla m_n}{m_e} = \frac{m_{nR}}{m_e} - \frac{1}{2}\beta_n = 1.53099(17) \quad (7)$$

Substituting  $\frac{m_{nR}}{m_e}$  from Equation (7) into Equation (5) gives the normalized energy equation in terms of the single variable  $\beta_n$  as

$$\frac{\nabla m_n}{m_e} = \frac{1}{\sqrt{1-\beta_n}} - \beta_n - 1 = 1.53099(17) \quad (8)$$

The solution is found by iteration to be

$$\beta_n = 0.915829(09) \quad (9)$$

The value of  $V_{eno}$  is calculated from  $\beta_n = \frac{V_{eno}^2}{c^2}$  to obtain

$$V_{eno} = c\sqrt{\beta_n} = 2.868982(11) \times 10^8 \text{ m/s} \quad (10)$$

Postulate No. 7 above states that Coulomb's law must be modified for the inner orbits nearest the proton by changing the point source for the electrostatic force to a source located at the fully contracted electron or proton shell. Therefore, the force balance equation for the neutron inner orbit model is the electron centripetal force equated to the modified Coulomb's law for the electrostatic force resulting in

$$\frac{m_e V_{eno}^2}{r_n} = \frac{k_e e^2}{r_{esn}^2} \quad (11)$$

The  $r_n$  is the electron  $n^{\text{th}}$  orbital radius,  $r_{esn}$  is the electrostatic force radius for the  $n^{\text{th}}$  orbit,  $k_e$  is the Coulomb constant, and  $e$  is the elementary charge.

It is important to note that the centripetal force involves only the constant electron mass although the orbital speed is near the speed of light. Under these conditions, the electron is accompanied by a significant relativistic mass, relativistic mee, and relativistic linear momentum due to the compression of the C2 particle grid system in the forefront of the electron<sup>2</sup>. However, the accompanying relativistic mass is not a part of the electron structure, and therefore does not influence the electron centripetal force.

The geometric relationship between  $r_n$  and  $r_{esn}$  is

$$r_n = r_{esn} + r_{pi} \quad (12)$$

The radius of the fully contracted proton shell<sup>1</sup> is  $r_{pi} = 1.2685188271(13) \times 10^{-15} \text{m}$ . Multiplying Equation (11) by  $r_n$  and dividing by  $k_e e^2$  gives

$$\frac{r_n}{r_{esn}^2} = \frac{m_e V_{eno}^2}{k_e e^2} = \frac{m_e \beta_n c^2}{k_e e^2} = \gamma_n \quad (13)$$

Combining Equations (12) and (13), the equation becomes

$$\gamma_n r_{esn}^2 - r_{esn} - r_{pi} = 0 \quad (14)$$

This quadratic equation is solved to obtain

$$r_{esn} = \frac{1 + \sqrt{1 + 4\gamma_n r_{pi}}}{2\gamma_n} \quad (15)$$

The value of  $\gamma_n$  is calculated from Equation (13) using the value of  $\beta_n$  from Equation (9) to obtain

$$\gamma_n = \frac{m_e \beta_n c^2}{k_e e^2} = 3.249994(32) \times 10^{14} \quad (16)$$

The value of  $r_{esn}$  is calculated from Equation (15) as

$$r_{esn} = 4.042464(47) \times 10^{-15} \text{ m} \quad (17)$$

The value of  $r_n$  is calculated from Equation (12) as

$$r_n = 5.310983(47) \times 10^{-15} \text{ m} \quad (18)$$

It is recognized that the assumption of circular orbits results in larger uncertainties than those shown in the above calculated values. This is because the value of the inner orbit radius given by Equation (18) is only a factor of 4.2 greater than the proton radius. At this close distance the electron's orbital motion will cause some associated motion in the proton. As a result the inner orbit will be somewhat elliptic relative to the proton. A conservative evaluation of the uncertainties resulting from the circular orbit assumption was made using an approximate analysis for an elliptic orbit. The reduced mass  $\mu$  for the electron/proton system was calculated to be

$$\mu = \frac{m_p m_e}{m_p + m_e} = 9.1044245(12) \times 10^{-31} \text{ kg} \quad (19)$$

It is noted that the difference between  $\mu$  and  $m_e$  is only 0.05443 percent, suggesting that the actual orbit is only slightly elliptic. The above calculations were repeated using the reduced mass instead of the electron mass. The new uncertainties were taken to be the differences in the extremes of the two calculations for each parameter. The new values and uncertainties are

$$r_{esn} = 4.0425(19) \times 10^{-15} \text{ m} \quad (20)$$

$$r_n = 5.3110(19) \times 10^{-15} \text{ m} \quad (21)$$

The new value and uncertainty for  $V_{eno}$  is calculated from Equation (11) as

$$V_{eno} = \left[ \frac{k_e e^2 r_n}{m_e r_{esn}^2} \right]^{1/2} = 2.8690(19) \times 10^8 \text{ m/s} \quad (22)$$

The new value and uncertainty for  $\beta_n$  becomes

$$\beta_n = \frac{V_{eno}^2}{c^2} = 0.9158(12) \quad (23)$$

Postulate No. 2 above states that the inner orbits are inside of the Bohr radius with values of angular momentum quantized by inverse integers. The quantized momentum equation is equivalent to the equation used in the analysis of the Bohr atom<sup>3</sup> but using inverse integers instead of direct integers to obtain

$$L_n = m_e V_{eno} r_n = n^{-1} \frac{h}{2\pi} \quad n = 1, 2, 3, \dots \quad (24)$$

The  $L_n$  is the angular momentum for the  $n^{\text{th}}$  orbit, and  $h$  is the Planck constant. Solving this equation for the orbital index number with its associated uncertainty due to the circular orbit assumption gives

$$n = \frac{h}{2\pi m_e V_{eno} r_n} = 75.977(78) \quad (25)$$

It is recognized that these calculated neutron properties are actually average values since the electron orbit is slightly elliptic resulting in a variable orbital radius. The uncertainty analysis showed that these deviations from the constant values for the circular orbit assumption are properly accounted for in the conservatively adjusted uncertainties. It is noted that under these circumstances, the orbital index number also has an uncertainty because of the variable radius orbit.

These calculated neutron properties are seen to be both physically realistic and meaningful. The electron orbital index number is an integer within the evaluated uncertainty. A very important observation from this analysis is that the neutron orbiting electron occupies the innermost possible orbit of  $n = 76$ . This is because  $n = 77$  corresponds to an orbital velocity of  $3.06366 \times 10^8 \text{ m/s}$  or  $\frac{V_{eno}}{c} = 1.0219$  which exceeds the speed of light and is therefore impossible for a structured particle. This observation indicates why an inner orbit electron cannot crash into the positively attractive proton. It also suggests that all inner orbiting electrons within atoms will occupy their innermost possible orbit.

The inner electron orbital radius which is 4.2 times the proton fully compressed radius is also physically realistic. Under the conditions for this special case, the expanded inner electron shares approximately 46 percent of its mass with the contracted proton consistent with the discussion for Postulate No. 4 above. The equation for the shared mass  $m_{ns}$  for  $n = 76$  is

$$\frac{m_{ns}}{m_e} = \frac{1}{2} \frac{V_{eno}^2}{c^2} = \frac{\beta_n}{2} = 0.45790(60) \quad (26)$$

This means that the center of the expanded electron is near but outside of the fully compressed proton radius.

It is noted that the inner electrons for atoms with two or more neutrons may not be orbiting because multiple electrons provide a repulsive force that can possibly balance the attractive electrostatic force. This is also the case for the outer electrons with two or more protons in the nucleus. The overall dynamic force balance for multiple protons and neutrons in the nucleus becomes a difficult mathematical problem. However, the recognition that the neutron is a proton with an inner orbit electron comparable to the proton with an outer orbit electron provides a more realistic approach to the analysis of atoms with multiple nucleons.

### General Case for the Neutron Inner Orbit Model

The general case for the neutron inner orbit model will utilize the orbital index number as the input parameter to determine the neutron properties for each possible orbit. This is in contrast with the special case above which utilized the known neutron excess  $m_e$  as the input parameter to determine the neutron properties including the orbital index number.

The force balance equation equates the electron centripetal force to the proton electrostatic force at all orbital locations using the modified Coulomb's law as in the special case above to obtain

$$\frac{m_e V_{eno}^2}{r_n} = \frac{k_e e^2}{r_{esn}^2} \quad (11)$$

The electron angular momentum is quantized as above using inverse orbital index integers  $n^{-1}$  to obtain

$$L_n = m_e V_{eno} r_n = n^{-1} \frac{h}{2\pi} \quad n = 1, 2, 3, \dots \quad (24)$$

The geometric relationship between  $r_n$  and  $r_{esn}$  also remains the same as

$$r_n = r_{esn} + r_{pi} \quad (12)$$

For a specified value of  $n$ , these are three equations in the three unknowns  $V_{eno}$ ,  $r_n$  and  $r_{esn}$ .

Eliminating  $r_n$  by substituting  $r_n = \frac{m_e V_{eno}^2}{k_e e^2} r_{esn}^2$  from Equation (11) into the other two equations leaves

$$\frac{m_e^2 V_{eno}^3}{k_e e^2} r_{esn}^2 = n^{-1} \frac{h}{2\pi} \quad (27)$$

$$\frac{m_e V_{eno}^2}{k_e e^2} r_{esn}^2 = r_{esn} + r_{pi} \quad (28)$$

Eliminating  $r_{esn}$  from these last two equations leaves

$$\frac{h}{2\pi n m_e V_{eno}} = \left[ \frac{h k_e e^2}{2\pi n m_e^2 V_{eno}^3} \right]^{1/2} + r_{pi} \quad (29)$$

This equation can be rewritten as

$$nr_{pi}V_{eno}^{3/2} - V_{eno}^{1/2}b_1 + n^{1/2}b_2 = 0 \quad (30)$$

Where

$$b_1 = \frac{h}{2\pi m_e} = 1.15767636(10) \times 10^{-4} \text{ m}^2/\text{s} \quad (31)$$

$$b_2 = \left[ \frac{hk_e e^2}{2\pi m_e^2} \right]^{1/2} = 1.71230128(15) \times 10^{-1} \text{ m}^{5/2}/\text{s}^{3/2} \quad (32)$$

$$r_{pi} = 1.2685188271(13) \times 10^{-15} \text{ m} \quad (33)$$

In order to solve this equation for the remaining unknown variable  $V_{eno}$  let

$$x = V_{eno}^{1/2} \quad (34)$$

$$x^3 = V_{eno}^{3/2} \quad (35)$$

Now divide Equation (30) by  $nr_{pi}$  to obtain

$$x^3 + ax + b = 0 \quad (36)$$

Where

$$a = -\frac{b_1}{nr_{pi}} = -\left(\frac{1}{n}\right) 9.12620558(99) \times 10^{10} \text{ m/s} \quad (37)$$

$$b = -\frac{b_2}{n^{1/2}r_{pi}} = -\left(\frac{1}{n^{1/2}}\right) 1.34984302(15) \times 10^{14} \text{ m}^{3/2}/\text{s}^{3/2} \quad (38)$$

The solution to the cubic equation is

$$x = 2\sqrt{-\frac{a}{3}} \cos(\theta + 240^\circ) \quad (39)$$

Where  $\theta$  in degrees is

$$\theta = \frac{1}{3} \cos^{-1} \left[ \frac{-b}{2\left(\frac{a^3}{27}\right)^{1/2}} \right] \quad (40)$$

The expression for  $V_{eno}$  in m/s becomes

$$V_{eno} = x^2 = \left[ 2\sqrt{-\frac{a}{3}} \cos(\theta + 240^\circ) \right]^2 \quad (41)$$

Substituting the above values for a and b into Equations (40) and (41) gives

$$\theta = \frac{1}{3} \cos^{-1} [-(n) 1.27203710(20) \times 10^{-2}] \quad (42)$$

$$V_{eno} = \left[ \frac{1}{n^{1/2}} [3.48830533(19) \times 10^5] \cos(\theta + 240^\circ) \right]^2 \quad (43)$$

Having derived the equation for  $V_{eno}$ , the expression for  $r_{esn}$  is developed as in the special case in Equations (12) – (15) to obtain

$$r_{esn} = \frac{1 + \sqrt{1 + 4\gamma_n r_{pi}}}{2\gamma_n} \quad (15)$$

Where

$$\gamma_n = \frac{m_e V_{eno}^2}{k_e e^2} \quad (13)$$

The  $r_n$  is then calculated from the geometric relationship

$$r_n = r_{esn} + r_{pi} \quad (12)$$

The value of  $\beta_n$  is found from

$$\beta_n = \frac{V_{eno}^2}{c^2} \quad (23)$$

The uncertainties shown in the above numerical values for this general case are properly called the precision of the calculations. The precision arises solely from the uncertainties in  $h$ ,  $m_e$ ,  $e$ , and  $k_e$ . It does not address the accuracy of the numerical value, which is the degree of closeness to the established experimentally value. Conservative limits for the numerical values will be established by assigning the relative standard uncertainties found in the special case above to the same parameters calculated in the general case. These parameters and their relative standard uncertainties  $u_r$  are

$$\begin{aligned} V_{eno} \quad u_r &= 6.6 \times 10^{-4} \\ r_{esn} \quad u_r &= 4.7 \times 10^{-4} \\ r_n \quad u_r &= 3.6 \times 10^{-4} \\ \beta_n \quad u_r &= 1.3 \times 10^{-3} \end{aligned}$$

The general case will now be solved using the same orbital index number with its uncertainty that was obtained in the special case which is

$$n = 75.977(78) \quad (25)$$

The values of  $\theta$  and  $V_{eno}$  are calculated from Equations (42) and (43) to obtain  $\theta = 55.0392^\circ$  and  $V_{eno} = 2.8689(19) \times 10^8 \text{ m/s}$  where the relative standard uncertainty from the special case has been used as discussed above. Likewise, Equations (12), (13) and (15) are solved to obtain  $r_{esn} = 4.0425(19) \times 10^{-15} \text{ m}$  and  $r_n = 5.3110(19) \times 10^{-15} \text{ m}$ . Equation (23) is solved to obtain  $\beta_n = 0.9158(12)$ . These values are the same as for the special case which validates the equations developed for the general case. Now the normalized neutron mee is calculated from Equation (8) using the new  $\beta_n$  to obtain

$$\frac{\nabla m_n}{m_e} = \frac{1}{\sqrt{1 - \beta_n}} - \beta_n - 1 = 1.530(24) \quad (8)$$

This value is equivalent to the experimentally determined value of 1.53099(17) to within the uncertainty of the calculation.

### **Summary**

The results for both the special case, in which the input is the known neutron excess mass equivalent energy, and the general case, in which the input is the orbital index number, produces the same values for the physical properties of the neutron inner orbit model. The compatible models for the electron and proton developed in Ref. 1 also satisfy the experimental results in the form of the fundamental constants and the electron and proton properties. The combination of these successful analyses implies that the principle of emergence upon which they are based may have even broader implications for all processes in the universe. This possibility is pursued in more detail in Ref. 2 entitled, “The Emergent Universe”.

### **References**

1. Callens G. The Physical Electron. <http://genecallens.net> (2013).
2. Callens G. The Emergent Universe. <http://genecallens.net> (2013).
3. Cutnell, J.D. & Johnson, K.W. Physics (John Wiley & Sons, Inc., 1998).

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